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PROPAGATION OF THE SOUND WAVE BY AN UNCLOSED SPHERICAL SHELL AND A PENETRABLE ELLIPSOID

In this paper the result of solution of axisymmetric problem of propagation of sound wave by an unclosed spherical shell and a penetrable ellipsoid of rotation is presented. A spherical radiator is located in a thin unclosed spherical shell as a source of acoustic field. The equation of the spheroidal boundary is given in spherical coordinates. A scattered pressure field is expressed in terms of spherical wave functions. Using corresponding additional theorems the solution of boundary value problem is reduced to solving of dual equations in Legendre's polynomials, which are converted to infinite system of linear algebraic equations of the second kind. The formula for calculation of the far field and numerical results for different values of parameters are obtained.

Keywords: sound field, spherical shell, ellipsoid of rotation, dual equations, spherical radiator

1. Introduction

The study of sound waves propagation in different media has a number of practical applications in electroacoustics, hydroacoustics, medical diagnostics, bioacoustics, creation of multi-layer sound-absorbing panels against noise and vibration [1-4]. Numerous publications describe the problems of propagation of sound field by different objects and they use different analytical and numerical techniques. We will consider just a few publications related to the research topic. The propagation of the sound field by hard or soft, prolate or oblate spheroids using different techniques is considered in [5-12]. The results of the sound field propagation on permeable and elastic spheroids are studied in [13-17]. The analytical description of the acoustic field scattered by a inhomogeneous elastic spheroid is obtained in [18]. The analytical solution to the diffraction problem of the plane sound wave on an elastic spheroid with arbitrary located spherical cavity is constructed in [19].

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In this paper analytical solution to axisymmetric problem of propagation of the sound wave by an unclosed thin spherical shell and a penetrable ellipsoid of rotation is presented. A spherical radiator is located in a thin unclosed spherical shell as a source of acoustic field. The equation of the spheroidal boundary is given in spherical coordinates. The solution of boundary value problem is reduced to solving of dual equations in Legendre's polynomials which are converted to infinite system of linear algebraic equations of the second kind. Numerical results are given for various values of parameters of the problem.

2. Problem formulation

Let a homogeneous space R^3 contain a thin unclosed spherical shell Γ_1 located on a sphere Γ of radius d with the center at point O and an ellipsoid shell S (fig. 1). We denote by D_1 the area of space bounded by the sphere Γ and by D_3 the area of space bounded by the ellipsoid shell S then $D_2 = R^3 \setminus (D_1 \cup \Gamma \cup D_3 \cup S)$ holds. The distance between points O and O_1 is equal to h_1.

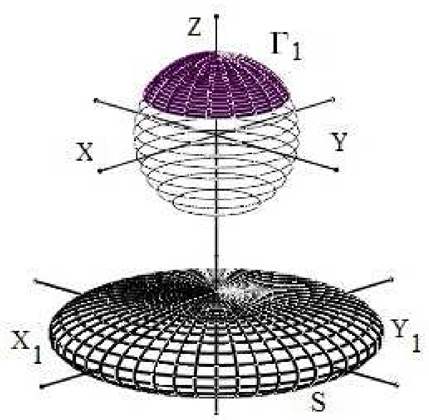

Fig. 1. Geometry of the problem

A point radiator of sound wave oscillating with an angular frequency ω is located at the point O. Areas D_j, $j=1,2,3$, are filled with material in which shear waves are not being spread. Let denote the density of medium by ρ_j and the speed of sound by c_j in D_j, $j=1,2,3$, $\rho_1 = \rho_2$.

To solve this problem we introduce spherical coordinates with the point O and the point O_1. The spherical shell Γ_1 and the ellipsoid shell S are described as follows:

$$\Gamma_1 = \{r = d,\ 0 \le \theta \le \theta_0 < \pi,\ 0 \le \varphi \le 2\pi\}, \tag{1}$$

$$S = \{r_1 = r(\theta_1),\ 0 \le \theta_1 \le \pi,\ 0 \le \varphi \le 2\pi\}, \tag{2}$$

where $r(\theta_1) = a / \sqrt{1 - v\sin^2\theta_1}$, $v = e^2/(e^2 - 1)$ stands for a prolate ellipsoid of rotation, $v = e^2$ is used for an oblate ellipsoid of rotation, e is the eccentricity of the ellipse.

Let p_c be a pressure of primary point radiator of sound field, p_j be a pressure of secondary sound field in the area D_j, $j=1,2,3$, then the real sound pressure is calculated by the formula $P_j = \mathrm{Re}\left(p_j e^{-i\omega t}\right)$.

Solution of the diffraction problem is reduced to finding pressures p_j, $j=1, 2, 3$, which satisfy:

1) the Helmholtz equation [20]

$$\Delta p_j + k_j^2 p_j = 0 \tag{3}$$

where $\Delta = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$ is the Laplace operator, $k_j = \omega / c_j$ is a wave number,

2) boundary condition on the surface of the spherical shell Γ_1 (acoustically hard shell):

$$\frac{\partial}{\partial \vec{n}}\left(p_c + p_1\right)\Big|_{\Gamma_1} = 0, \tag{4}$$

where $\vec{n}$ is the normal to the surface Γ_1,

3) boundary conditions on the surface of the ellipsoidal shell S:

$$p_2\big|_S = p_3\big|_S, \qquad \frac{1}{\rho_2}\frac{\partial}{\partial \vec{n}} p_2\bigg|_S = \frac{1}{\rho_3}\frac{\partial}{\partial \vec{n}} p_3\bigg|_S, \tag{5}$$

where $\vec{n}$ is the normal to the surface S and the condition at infinity [20-23]:

$$\lim_{M\to\infty} r\left(\frac{\partial p_2(M)}{\partial r} - i k p_2(M)\right) = 0, \tag{6}$$

where M is an arbitrary point at the space.

The condition of the continuity of pressure on the open part of the spherical shell $\Gamma \setminus \Gamma_1$ and the normal derivative on the surface of the sphere Γ are given by [22, 23]:

$$\left(p_c + p_1\right)\Big|_{\Gamma\setminus\Gamma_1} = p_2\Big|_{\Gamma\setminus\Gamma_1}, \qquad \frac{\partial}{\partial r}\left(p_c + p_1\right)\Big|_{\Gamma} = \frac{\partial}{\partial r} p_2\Big|_{\Gamma}. \tag{7}$$

3. Presentation of problem solution

The initial pressure of sound field can be presented in the form [21]:

$$p_c(r,\theta) = P\exp(ik_1 r)/r = P\sum_{n=0}^{\infty} f_n h_n^{(1)}\left(k_1 r\right) P_n(\cos\theta), \quad f_n = ik\delta_{0n} \tag{8}$$

where $h_n^{(1)}\left(x\right)$ are the spherical Hankel functions, $P_n(\cos\theta)$ are the Legendre polynomials, δ_{0n} is the Kronecker delta, P is constant.

The pressure of the scattered sound field presented as a superposition of the basic solutions of the Helmholtz equation in spherical coordinates taking into account the condition at infinity (6):

$$p_1(r,\theta) = P\sum_{n=0}^{\infty} a_n j_n\left(k_1 r\right) P_n\left(\cos\theta\right), \quad r < d, \tag{9}$$

$$p_2^{(1)}\left(r, \theta\right) = P\sum_{n=0}^{\infty} x_n h_n^{(1)}\left(k_1 r\right) P_n\left(\cos\theta\right), \quad r > d, \tag{10}$$

$$p_2^{(2)}\left(r_1, \theta_1\right) = P\sum_{n=0}^{\infty} y_n h_n^{(1)}\left(k_1 r_1\right) P_n\left(\cos\theta_1\right), \quad r_1 > r\left(\theta_1\right), \tag{11}$$

$$p_3(r_1,\theta_1) = P\sum_{n=0}^{\infty} b_n j_n\left(k_3 r_1\right) P_n\left(\cos\theta_1\right), \quad r_1 < r(\theta_1) \tag{12}$$

where $p_2 = p_2^{(1)}(r,\theta) + p_2^{(2)}(r_1,\theta_1)$, $j_n\left(x\right)$ are the spherical Bessel functions of the first kind.

Unknown coefficients a_n, b_n, x_n, y_n must be determined from the boundary conditions.

4. Fulfilment of boundary conditions

First, we will express function $p_2^{(2)}(r_1,\theta_1)$ through the spherical wave functions in the coordinate system with the origin at the point O using the formulae [22, 23]:

$$\left.\begin{aligned}
&h_n^{(1)}(kr_1)P_n(\cos\theta_1)=\sum_{k=0}^{\infty}A_{nk}(h_1)\,j_k(kr)P_k(\cos\theta),\quad r<h_1,\\
&A_{nk}(h_1)=(2k+1)\sum_{\sigma=|k-n|}^{k+n} i^{\sigma+k-n}\,b_\sigma^{(n0k0)}\,h_\sigma^{(1)}(kh_1),\\
&b_\sigma^{(n0q0)}=(nq00\,|\,\sigma 0)^2,\quad (nq00\,|\,\sigma 0)\ \text{are the Clebsch–Gordan coefficients,}
\end{aligned}\right\}\tag{13}$$

then

$$p_2^{(2)}(r,\theta)=P\sum_{n=0}^{\infty}p_n\,j_n(k_1r)P_n(\cos\theta),\ \ p_n=\sum_{k=0}^{\infty}y_kA_{kn}(h_1)\tag{14}$$

Considering (8)-(10), (14) and taking into account the condition of orthogonality of the Legendre polynomials on the interval $[0;\pi]$ the boundary conditions (4), (7) will become:

$$\left.\begin{aligned}
&\sum_{n=0}^{\infty}x_n\frac{d}{d\xi_0}h_n^{(1)}(\xi_0)P_n(\cos\theta)=-\sum_{n=o}^{\infty}p_n\frac{d}{d\xi_0}j_n(\xi_0)P_n(\cos\theta),\quad 0\le\theta<\theta_0,\\
&\sum_{n=0}^{\infty}\frac{x_n-f_n}{\frac{d}{d\xi_0}j_n(\xi_0)}P_n(\cos\theta)=0,\ \ \theta_0<\theta\le\pi,\ \ \xi_0=k_1d.
\end{aligned}\right\}\tag{15}$$

We introduce new coefficients X_n by the formula

$$x_n=X_n\frac{d}{d\xi_0}j_n(\xi_0)+f_n,\quad n=0,1,...\tag{16}$$

and the small parameter

$$g_n=1+\frac{4i\xi_0^3}{2n+1}\frac{d}{d\xi_0}j_n(\xi_0)\frac{d}{d\xi_0}h_n^{(1)}(\xi_0)\ .\tag{17}$$

From the asymptotic representations for functions $j_n(x), h_n^{(1)}(x)$, where $n\gg x$ [21]:

$$j_n(x) \approx \frac{2^n n!(x)^n}{(2n+1)!}, \quad h_n^{(1)}(x) \approx -\frac{i(2n)!}{2^n n!(x)^n}, \tag{18}$$

follows that $g_n = O\left(n^{-2}\right)$.

The dual series equations (15) take the form

$$\left.\begin{aligned} &\sum_{n=0}^{\infty}(2n+1)(1-g_n)X_n P_n(\cos\theta) = \sum_{n=o}^{\infty}(2n+1)(\tilde{f}_n+\tilde{p}_n)P_n(\cos\theta), \; 0 \le \theta < \theta_0, \\ &\sum_{n=0}^{\infty} X_n P_n(\cos\theta) = 0, \quad \theta_0 < \theta \le \pi, \end{aligned}\right\} \tag{19}$$

where

$$\tilde{f}_n = 4i\xi_0^3 f_n \frac{d}{d\xi_0} h_n^{(1)}(\xi_0)/(2n+1), \quad \tilde{p}_n = 4i\xi_0^3 a_n \frac{d}{d\xi_0} j_n(\xi_0)/(2n+1). \tag{20}$$

The dual series equations are transformed to the infinite system of linear algebraic equations of the second kind with the completely continuous operator using the integral representation for the Legendre polynomials [22]:

$$X_n - \sum_{k=0}^{\infty} g_k R_{nk}(\theta_0) X_k = \sum_{k=0}^{\infty}\left(\tilde{p}_k + \tilde{f}_k\right) R_{nk}(\theta_0), \quad n = 0, 1, \ldots, \tag{21}$$

where

$$R_{nk}(\theta_0) = \frac{1}{\pi}\left[\frac{\sin(n-k)\theta_0}{n-k} - \frac{\sin(n+k+1)\theta_0}{n+k+1}\right], \quad \left.\frac{\sin(n-k)\theta_0}{n-k}\right|_{n=k} = \theta_0 \,. \tag{22}$$

Now we present the function $p_2^{(1)}(r, \theta)$ through the spherical wave functions in the coordinate system with origin at the point O_1 using formula [22, 23]:

$$h_n^{(1)}(kr) P_n(\cos\theta) = \sum_{k=0}^{\infty} B_{nk}(h_1) j_k(kr_1) P_k(\cos\theta_1), \quad r_1 < h_1, \tag{23}$$

then

$$p_2^{(1)}\left(r_1,\,\theta_1\right)=P\sum_{n=0}^{\infty}z_n j_n(k_1 r_1)P_n\left(\cos\theta_1\right),\ z_n=\sum_{p=0}^{\infty}x_p B_{pn}\left(h_1\right), \tag{24}$$

where

$$B_{nk}(h_1)=\left(2k+1\right)\sum_{\sigma=|k-n|}^{k+n}(-1)^{\sigma}\,i^{\sigma+k-n}\,b_{\sigma}^{(n0k0)}\,h_{\sigma}^{(1)}(k_1 h_1)\,. \tag{25}$$

In view of the fact that

$$\left.\begin{aligned}&\left.\frac{\partial}{\partial\vec{n}}p_j(r_1,\theta_1)\right|_S=\frac{\partial}{\partial r_1}p_j(r_1,\theta_1)-\frac{r'(\theta_1)}{r(\theta_1)^2}\frac{\partial}{\partial\theta_1}p_j(r_1,\theta_1)\Bigg|_{r_1=r(\theta_1)},\quad j=2,3,\\&\frac{d}{d\theta_1}P_n(\cos\theta_1)=P_n^1(\cos\theta_1),\end{aligned}\right\} \tag{26}$$

boundary conditions (5) together with (11), (12), (24) take the form

$$\left.\begin{aligned}&\sum_{n=0}^{\infty}z_n j_n(k_1 r(\theta_1))P_n\left(\cos\theta_1\right)+\sum_{n=0}^{\infty}y_n h_n^{(1)}\left(k_1 r(\theta_1)\right)P_n\left(\cos\theta_1\right)=\\&=\sum_{n=0}^{\infty}b_n j_n\left(k_3 r(\theta_1)\right)P_n\left(\cos\theta_1\right),\end{aligned}\right\} \tag{27}$$

$$\left.\begin{aligned}&\sum_{n=0}^{\infty}z_n k_1 j_n'(\xi_1)P_n(\cos\theta_1)-\frac{r'(\theta_1)}{r(\theta_1)^2}j_n(\xi_1)P_n^1(\cos\theta_1)+\\&+\sum_{n=0}^{\infty}y_n k_1 h_n^{(1)\prime}(\xi_1)P_n(\cos\theta_1)-\frac{r'(\theta_1)}{r(\theta_1)^2}h_n^{(1)}(\xi_1)P_n^1(\cos\theta_1)\ =\\&=\frac{\rho_1}{\rho_3}\sum_{n=0}^{\infty}b_n k_3 j_n'(\xi_3)P_n(\cos\theta_1)-\frac{r'(\theta_1)}{r(\theta_1)^2}j_n(\xi_3)P_n^1(\cos\theta_1),\ \xi_j=k_j r(\theta_1),\ j=1,3.\end{aligned}\right\} \tag{28}$$

Let eliminate coefficients z_n in (27), (28) using (24), (16). We multiply the resulting equations by $P_s\left(\cos\theta\right)\sin\theta d\theta$, $s=0,1,2,...$, and integrate from 0 to π, and link with (21) then we have

$$\left.\begin{aligned}
&\sum_{n=0}^{\infty}(g_n R_{sn}(\theta_0)-\delta_{ns})X_n+\sum_{n=0}^{\infty}\tilde{b}_{ns}(k_1)y_n=4\xi_0^3 k_1\frac{d}{d\xi_0}h_0^{(1)}(\xi_0)R_{s0}(\theta_0), s=0,1,2,...,\\
&\sum_{n=0}^{\infty}X_n\tilde{a}_{ns}(k_1)+\sum_{n=0}^{\infty}y_n b_{ns}(k_1)-\sum_{n=0}^{\infty}b_n a_{ns}(k_3)=-ik_1\sum_{n=0}^{\infty}B_{0n}(h_1)a_{ns}(k_1),\\
&\sum_{n=0}^{\infty}X_n\tilde{A}_{ns}(k_1)+\sum_{n=0}^{\infty}y_n B_{ns}(k_1)-\frac{\rho_3}{\rho_1}\sum_{n=0}^{\infty}b_n A_{ns}(k_3)=-ik_1\sum_{n=0}^{\infty}B_{0n}(h_1)A_{ns}(k_1),
\end{aligned}\right\}\qquad(29)$$

where

$$\left.\begin{aligned}
&a_{n,s}(k_j)=\int_0^{\pi}j_n(\xi_j)P_n(\cos\theta_1)P_s(\cos\theta_1)\sin\theta_1 d\theta_1.\\
&b_{n,s}(k_1)=\int_0^{\pi}h_n^{(1)}(\xi_j)P_n(\cos\theta_1)P_s(\cos\theta_1)\sin\theta_1 d\theta_1,\\
&\tilde{a}_{ns}(k_j)=\frac{d}{d\xi_0}j_n(\xi_0)\sum_{m=0}^{\infty}a_{ms}(k_j)B_{nm}(h_1),\quad j=1,3,\\
&\tilde{b}_{ns}(k_1)=4i\xi_0^3\sum_{p=0}^{\infty}\frac{d}{d\xi_0}j_p(\xi_0)R_{sp}(\theta_0)A_{np}(h_1)/(2p+1),\\
&\alpha_{n,s}(k_j)=\int_0^{\pi}\frac{d}{d\xi_j}j_n(\xi_j)P_n(\cos\theta_1)P_s(\cos\theta_1)\sin\theta_1 d\theta_1,\\
&\beta_{n,s}(k_1)=\int_0^{\pi}\frac{d}{d\xi_1}h_n^{(1)}(\xi_1)P_n(\cos\theta_1)P_s(\cos\theta_1)\sin\theta_1 d\theta_1,\\
&\tilde{\alpha}_{n,s}(k_j)=\int_0^{\pi}\frac{r'(\theta_1)}{r(\theta_1)^2}j_n(\xi_j)P_n^1(\cos\theta_1)P_s(\cos\theta_1)\sin\theta_1 d\theta_1,\\
&\tilde{\beta}_{n,s}(k_1)=\int_0^{\pi}\frac{r'(\theta_1)}{r(\theta_1)^2}h_n^{(1)}(\xi_1)P_n^1(\cos\theta_1)P_s(\cos\theta_1)\sin\theta_1 d\theta_1,\\
&A_{n,s}(k_j)=k_j\alpha_{n,s}(k_j)-\tilde{\alpha}_{n,s}(k_j),\quad B_{n,s}(k_1)=k_1\beta_{n,s}(k_1)-\tilde{\beta}_{n,s}(k_1),\\
&\tilde{A}_{n,s}(k_1)=\frac{d}{d\xi_0}j_n(\xi_0)\sum_{m=0}^{\infty}A_{ms}(k_1)B_{nm}(h_1).
\end{aligned}\right\}\qquad(30)$$

The infinite system (28) can be solved by the method of truncation [21].

5. Numerical experiments

Based on [23]:

$$\left.\begin{aligned} & h_n^{(1)}(kr_1)P_n(\cos\theta_1)=\sum_{p=0}^{\infty}\tilde{A}_{np}(h_1)h_p^{(1)}(kr)P_p(\cos\theta),\ r>h_1, \\ & \tilde{A}_{np}(h_1)=\sum_{\sigma=|p-n|}^{p+n}(2\sigma+1)i^{\sigma+p-n}\,b_p^{(n0\sigma0)}\,j_\sigma(kh_1) \end{aligned}\right\} \qquad (31)$$

we present the function $p_2^{(2)}(r_1,\theta_1)$ in the coordinate system at the point O:

$$p_2^{(2)}(r,\theta)=P\sum_{n=0}^{\infty}N_n h_n^{(1)}(kr)P_n(\cos\theta),\quad N_n=\sum_{p=0}^{\infty}\tilde{A}_{pn}(h_1)y_p \qquad (32)$$

Using the asymptotic expression for the function $h_n^{(1)}(kr)$ [21]:

$$h_n^{(1)}(kr)\approx(-i)^{n+1}e^{ikr}/kr,\quad kr\to\infty \qquad (33)$$

we obtain the pressure in the far zone:

$$p_2(r,\theta)=P\frac{e^{ikr}}{kr}G(\theta)\,, \qquad (34)$$

where

$$G(\theta)=\sum_{n=0}^{\infty}(-i)^{n+1}\left(X_n\frac{d}{d\xi_0}j_n(\xi_0)+f_n+\sum_{p=0}^{\infty}\tilde{A}_{pn}(h_1)y_p\right)P_n(\cos\theta)\,. \qquad (35)$$

The unknown coefficients X_n, y_p are found from the system (29). Using Mathcad [24] the function G(θ) has been calculated for some parameters of the problem. The spherical functions have been calculated by means of built-in functions Mathcad. The infinite system (36) has been solved by the method of truncation [21]. The computational experiment showed that the truncation order for the considered parameters of the problem can be equal to 25. With this truncation the solution of the system (29) has accuracy 10^{-4}.

Figure 2 shows plots of function G(θ) for some values of the frequency f of the sound field. The remaining parameters are equal to: h_1 = 1.0 m, a = 0.3 m, b = 2a, d = 0.2 m, $\theta_0=\pi/4$. The areas D_1, D_2 are filled with water ($c_1=c_2$ = 1483 m/s, $\rho_1=\rho_2$ = 1000 kg/m^3). The area D_3 is filled with organic glass (c_3 = 2565 m/s, ρ_3 = 1200 kg/m^3).

Figure 3 shows plots of function $G(\theta)$ for some values of the angle θ_0 of the thin unclosed spherical shell Γ_1. The remaining parameters are equal to: $h_1 = 0.8$ m, $a = 0.3$ m, $b = 0.4$ m, $d = 0{,}1$ m, $f = 1000$ Hz. The areas D_1, D_2 are filled with water ($c_1 = c_2 = 1483$ m/s, $\rho_1 = \rho_2 = 1000$ kg/m^3). The area D_3 is filled with ice ($c_3 = 3980$ m/s, $\rho_3 = 900$ kg/m^3).

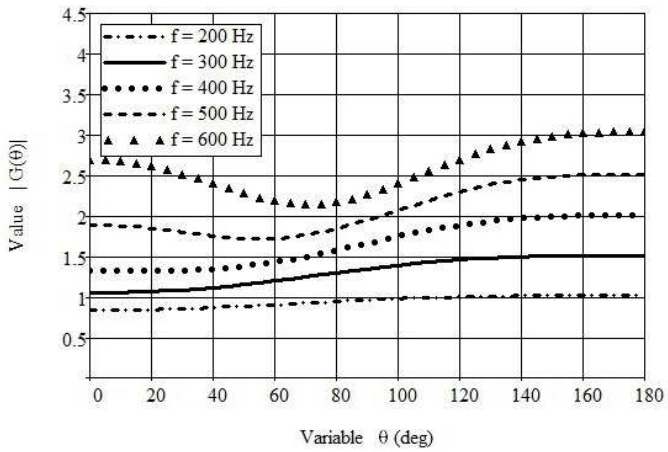

Fig. 2. Graphs of function $G(\theta)$ for some values of the frequency f of the sound field

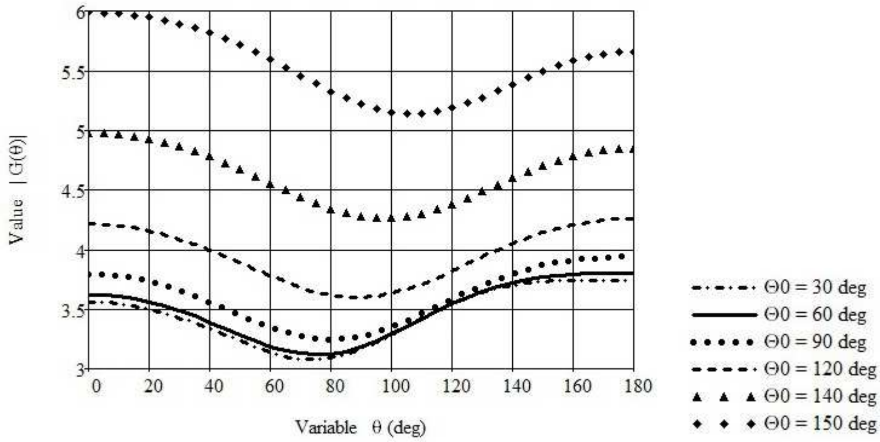

Fig. 3. Graphs of function $G(\theta)$ for some values of the angle θ_0

6. Conclusions

It has been shown that the solution of the problem of propagation of sound field by an unclosed spherical shell and a penetrable ellipsoid of rotation is reduced to the infinite system of linear algebraic equations of the second kind. The equation of spheroidal boundary is considered in the spherical coordinates. The spherical radiator is considered as the source of the sound field located within the thin unclosed spherical shell. The addition theorems for spherical wave functions and the method of solution of the dual series equations in Legendre's polynomials have been used. The developed methodology can be practically used in sound screen production.

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ROZCHODZENIE SIĘ FALI AKUSTYCZNEJ PRZEZ OTWARTĄ SFERYCZNĄ POWŁOKĘ ORAZ PRZENIKALNĄ ELIPSOIDĘ

S t r e s z c z e n i e

W artykule przedstawiono wynik rozwiązania problemu osiowosymetrycznego rozchodzenia się fali akustycznej przez otwartą sferyczną powłokę oraz przenikalną elipsoidę ruchu obrotowego. Sferyczny radiator, jako źródło pola akustycznego umieszczono w cienkiej sferycznej powłoce. Równanie granicy sferoidalnej podano we współrzędnych sferycznych. Pole rozproszonego ciśnienia wyrażono w funkcjach fal sferycznych. Wykorzystując odpowiednie dodatkowe twierdzenia rozwiązanie problemu wartości granicznej zredukowano do rozwiązania podwójnych równań w wielomianach Legendre'a, które przetworzono do systemu nieskończonego liniowych równań algebraicznych drugiego rodzaju. Otrzymano wzór do obliczenia pola przestrzennego oraz wyniki numeryczne dla różnych wartości parametrów.

Słowa kluczowe: pole akustyczne, powłoka sferyczna, elipsoida ruchu obrotowego, równania podwójne, radiator sferyczny

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